



# LOYOLA COLLEGE (AUTONOMOUS) CHENNAI – 600 034

U.G. DEGREE EXAMINATION – ALLIED  
FIRST SEMESTER – NOVEMBER 2024



## UMT 1301 – MATHEMATICS FOR PHYSICS, MATHEMATICS FOR PHYSICS - I

Date: 20-11-2024

Dept. No.

Max. : 100 Marks

Time: 09:00 am-12:00 pm

### SECTION A - K1 (CO1)

Answer ALL the Questions -

(10 x 1 = 10)

#### 1. Answer the following

a) If  $y = e^{ax}$ , find the  $n^{th}$  derivative.

b) Write the D'Alembert's ratio test.

c) State Cayley-Hamilton Theorem.

d) Find  $L[t^3]$ .

e) Define normal distribution.

#### 2. Fill in the blanks

a) The polar subnormal for the curve  $r = ae^{\theta \cot \alpha}$  is \_\_\_\_\_.

b) The expansion of  $\log(1+x)$ , when  $-1 < x < 1$  is \_\_\_\_\_.

c) The determinant of an  $n \times n$  identity matrix is \_\_\_\_\_.

d)  $L^{-1}\left[\frac{1}{s^2}\right] = \underline{\quad \quad}$

e) If a poisson variate  $X$  is such that  $P(X=1) = 2P(X=2)$ , then mean is \_\_\_\_\_.

### SECTION A - K2 (CO1)

Answer ALL the Questions

(10 x 1 = 10)

#### 3. Choose the correct answer.

a) The  $n^{th}$  derivative of  $y = \sin(ax+b)$  is

i)  $a^n \cos\left(\frac{n\pi}{2} + x\right)$

ii)  $a^n \sin\left(\frac{n\pi}{2} + x\right)$

iii)  $a^{n+1} \sin\left(\frac{n\pi}{2} + x\right)$

iv) None of above

b)  $(1-x)^{-1} = \underline{\quad \quad}$

i)  $1-x+x^2+\dots$

ii)  $1+x+x^2+\dots$

iii)  $1-x-x^2-\dots$

iv)  $1+x-x^2+\dots$

c) The eigen value of the matrix  $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 6 \\ 0 & 0 & 7 \end{bmatrix}$  is

i) 1,2,3

ii) 2,3,6

iii) 3,6,7

iv) 1,4,7

d) The inverse Laplace transform of  $\frac{1}{(s+1)^2+1}$  is

i)  $e^{-t} \cos t$

ii)  $e^{-t} \sin t$

iii)  $e^t \cos t$

iv)  $e^t \sin t$

e) Poisson distribution is defined by

i)  $\frac{e^\lambda \lambda^x}{x!}$

ii)  $\frac{e^{-\lambda} \lambda^x}{x!}$

iii)  $\frac{e^{-\lambda} \lambda^{-x}}{x!}$

iv)  $\frac{e^\lambda \lambda^{-x}}{x!}$

#### 4. True or False

a) The maximum value of  $f(x) = x^2$  is 0.

b)  $\log \frac{1+x}{1-x} = 2 \left( x + \frac{x^3}{3} + \frac{x^5}{5} + \dots \right)$

|    |                                                                                                        |
|----|--------------------------------------------------------------------------------------------------------|
| c) | The matrix $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 5 & 6 \end{bmatrix}$ is an symmetric matrix. |
| d) | $L(t^n f(t)) = (-1)^n \frac{d^n}{dt^n} [L(f(t))]$                                                      |
| e) | The mean and variance of the poison distribution are the same.                                         |

### SECTION B - K3 (CO2)

Answer any TWO of the following

(2 x 10 = 20)

5. Find  $y_n$ , where  $y = \frac{3}{(x+1)(2x-1)}$
6. Calculate the sum of the series to infinity using binomial series expansion  
 $\frac{1.4}{5.10} - \frac{1.4.7}{5.10.15} + \frac{1.4.7.10}{5.10.15.20} + \dots$
7. Find the characteristic roots of the matrix  $A = \begin{bmatrix} 2 & -2 & 3 \\ 1 & 1 & 1 \\ 1 & 3 & -1 \end{bmatrix}$ .
8. Find  $L^{-1} \left[ \frac{1}{S(S+1)(S+2)} \right]$ .

### SECTION C – K4 (CO3)

Answer any TWO of the following

(2 x 10 = 20)

9. Find the angle of intersection of the cardioids  $r = a(1 + \cos \theta)$  and  $r = b(1 - \cos \theta)$ .
10. Calculate the sum of the series  $\sum_{n=1}^{\infty} \frac{n^2+3}{n+2} \cdot \frac{x^n}{n!}$ ,
11. Solve the system of equations using crammer's rule.  
 $2x - y + 3z = 9, x + y + z = 6, x - y + z = 2$ .
12. Calculate the correlation coefficient for the following data.

|   |    |    |    |    |    |    |    |    |
|---|----|----|----|----|----|----|----|----|
| X | 65 | 66 | 67 | 67 | 68 | 69 | 70 | 72 |
| Y | 67 | 68 | 65 | 68 | 72 | 72 | 69 | 71 |

### SECTION D – K5 (CO4)

Answer any ONE of the following

(1 x 20 = 20)

13. a) If  $y = \sin(m \sin^{-1} x)$ , prove that  $(1-x^2)y_{n+2} - (2n+1)x y_{n+1} + (m^2 - n^2)y_n = 0$ .  
b) Sum the series  $1 + \frac{1+3}{2!} + \frac{1+3+3^2}{3!} + \frac{1+3+3^2+3^3}{4!} + \dots \infty$ .  
(10+10)
14. a) Using Laplace transform evaluate  $\int_0^{\infty} t e^{-3t} \cos t dt$ .  
b) Find the mean and standard deviation for the following data.

|   |    |    |    |    |    |     |
|---|----|----|----|----|----|-----|
| x | 10 | 20 | 30 | 40 | 50 | 60  |
| f | 15 | 32 | 51 | 78 | 97 | 109 |

(10+10)

### SECTION E – K6 (CO5)

Answer any ONE of the following

(1 x 20 = 20)

15. a) Find the  $n^{\text{th}}$  differential coefficient of  $\cos x \cos 2x \cos 3x$ .  
b) Verify Cayley Hamilton theorem for the matrix  $A = \begin{bmatrix} 2 & 2 & 0 \\ 2 & 1 & 1 \\ -7 & 2 & -3 \end{bmatrix}$ .  
(10+10)

|     |                                                                                                                                                                                                                                                                       |
|-----|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| 16. | <p>a) Solve the differential equation using Laplace transform.</p> $\frac{d^2 y}{dx^2} + 2 \frac{dy}{dx} + 5y = 4e^{-4t}, \text{ given that } y(0) = y'(0) = 0.$ <p>b) Ten coins are thrown simultaneously. Find the probability of getting at least seven heads?</p> |
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(10+10)

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